Journal of Applied Mechanics and Technical Physics, Vol. 42, No. 1, pp. 87–90, 2001

ENERGY ADDITION TO A GAS IN A TURBULENT SUPERSONIC BOUNDARY LAYER

O. B. Larin and V. A. Levin[1]

UDC 532.526.4:536.2.24

The effect of the thermal action on a turbulent supersonic boundary-layer flow is studied numerically. It is shown that the friction force on an isothermal surface decreases significantly. The effectiveness of using a thermal source for decreasing friction is evaluated.

Drag reduction during the motion of bodies in the atmosphere refers to the most important aerodynamic problems. The use of external sources of energy for acting on the friction force is of interest.

Two methods of heat addition to a turbulent boundary layer were compared in [1]: heating of a certain part of the body surface with the remaining part of the surface being thermally insulated and the use of a bulk thermal source. It was found that, with an identical amount of heat added to the gas, bulk heating leads to a greater decrease in friction than surface heating.

Based on the results of [2], we study numerically the effect of the thermal action on a turbulent supersonic boundary-layer flow, the amount of heat added to the gas being varied within wide limits.

1. Formulation of the Problem and Method of Solution. We consider a plane supersonic flow around a cooled plate in the presence of a rectangular thermal source in the boundary layer. The system of averaged equations of turbulent motion of a perfect gas in the absence of external mass forces has the following form:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v_*}{\partial y} = 0, \qquad \rho u \frac{\partial u}{\partial x} + \rho v_* \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\Big(\mu_* \frac{\partial u}{\partial y}\Big),$$

$$\rho u \frac{\partial I_0}{\partial x} + \rho v_* \frac{\partial I_0}{\partial y} = \frac{\partial}{\partial y}\Big(\lambda_* \frac{\partial T}{\partial y} + \mu_* u \frac{\partial u}{\partial y}\Big) + \rho Q, \tag{1.1}$$

$$\rho = \frac{pm}{RT}, \quad \rho v_* = \rho v + \langle \rho' v' \rangle, \quad I_0 = I + \frac{u^2}{2}, \quad \mu_* = \mu + \mu_{\rm t}, \quad \lambda_* = \Big(\frac{\mu}{\rm Pr} + \frac{\mu_{\rm t}}{\rm Pr_t}\Big) c_p.$$

Here u and v are the projections of the velocity vector onto the axes of orthogonal coordinates x (along the surface) and y (normal to it), respectively, ρ is the density, p is the pressure, T is the temperature, I is the enthalpy, $Q = Q(x, y)$ is the specific heat added from outside to a given point of the medium per unit time, m is the molecular weight of the gas, R is the universal gas constant, $\langle \rho' v' \rangle$ is the correlation of density fluctuations and the normal component of velocity, μ and $\mu_{\rm t}$ are the dynamic molecular and turbulent viscosities, c_p is the specific heat capacity of the gas at constant pressure, and Pr and $\rm Pr_t$ are the Prandtl number and its turbulent analog, respectively; three last parameters are assumed to be constant.

In the present work, the turbulent viscosity is determined using the two-layer model of Cebeci and Smith [3], which describes heat transfer in the boundary layer at a supersonic free-stream velocity. This

Institute of Mechanics at the Moscow State University, Moscow 119899. [1]Institute of Automation and Control Processes, Far-East Division, Russian Academy of Sciences, Vladivostok 690041. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 42, No. 1, pp. 98–101, January–February, 2001. Original article submitted August 9, 1999; revision submitted December 20, 1999.

0021-8944/01/4201-0087 $25.00 © 2001 Plenum Publishing Corporation

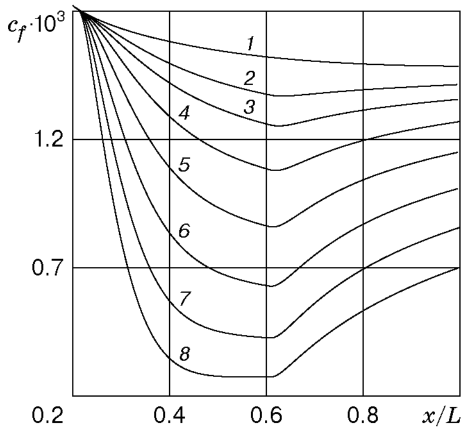

Fig. 1

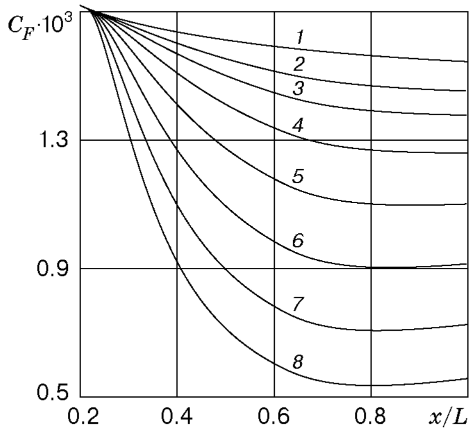

Fig. 2

model was tested on a rather wide class of problems. There is no sense in using a more complicated model because of the absence of experimental data on the flow considered.

The boundary conditions on the body surface $y = 0$ have the form $u = 0$, $v = 0$, and $T = T_w$. The flow characteristics at the external boundary of the layer marked below by the subscript e are assumed to be known.

The problem posed is solved numerically by the finite-difference method. System (1.1) is first transformed to a dimensionless form. We use the normal coordinate $\eta = \delta^{-1}(x) \int\limits_0^y \rho\, dy$, where $\delta(x)$ is the normalization function. We also use an unconditionally stable implicit difference scheme, which provides second-order approximation with respect to grid steps Δx and $\Delta\eta$ and stability of numerical calculations.

The difference relations for the derivatives of the sought functions in the streamwise direction have the form

$$\Big(\frac{\partial F}{\partial x}\Big)_m^{n+z} = \beta_z \frac{F_m^{n+z} - g_z}{x_{n+1} - x_n},$$

where n and m are the indices of grid nodes in the x and η directions, $\beta_z = 3$ and $g_z = F_m^n$ for $z = 1/3$, and $\beta_z = 4$ and $g_z = (9F_m^{n+1/3} - 5F_m^n)/4$ for $z = 1$. The solution in the layer $x = x_{n+1}$ is determined from the known values of the functions in the layer $x = x_n$ in two stages. At the first stage, it is found for $x = x_{n+1/3}$ ($z = 1/3$) with an error of order $O(\Delta x) + O((\Delta\eta)^2)$; at the second stage, the solution is found for $x = x_{n+1}$ ($z = 1$) with the main part of the approximation error $O((\Delta x)^2) + O((\Delta\eta)^2)$.

2. Results. We give some calculation results for a free-stream Mach number $\mathrm{M}_e = 3$. The temperature of the cooled wall is assumed to be equal to the gas temperature at the external boundary: $T_w = T_e$. The Prandtl numbers are $\mathrm{Pr} = 0.72$ and $\mathrm{Pr_t} = 0.9$, and the ratio of specific heats is $\gamma = 1.4$. For molecular viscosity μ, we use a power-law temperature dependence with an exponent $\omega = 0.76$.

It is assumed that the external heat addition with a constant parameter $q = Q(c_p T_e u_e)^{-1} L$ is performed in the rectangular region

$$0.2 < x/L \leqslant 0.6, \qquad 10 \leqslant y/Y \leqslant 30. \tag{2.1}$$

Here $Y = \mu_e \rho_e^{-1} u_e^{-1} \cdot 10^4$ and $L = Y \cdot 10^4$. Note that region (2.1) is located completely inside the boundary layer.

Figure 1 shows the distributions of the local skin-friction coefficient $c_f = 2\tau_w \rho_e^{-1} u_e^{-2}$, where $\tau_w = (\mu\, \partial u/\partial y)_{y=0}$ is the friction stress. Cures 1–8 in Figs. 1–4 correspond to the values of the specific heat addition $q = 0$, 1, 2, 4, 8, 16, 32, and 64, respectively. Energy release increases the boundary-layer thickness, displaces streamlines away from the plate, and decreases skin friction. It follows from Fig. 1 that the decrease in the local skin-friction coefficient is quite significant for the conditions considered. Thus, the local skin-friction coefficient at the end of the heat-release zone for $q = 64$ is 5.6 times lower than in the case $q = 0$. The

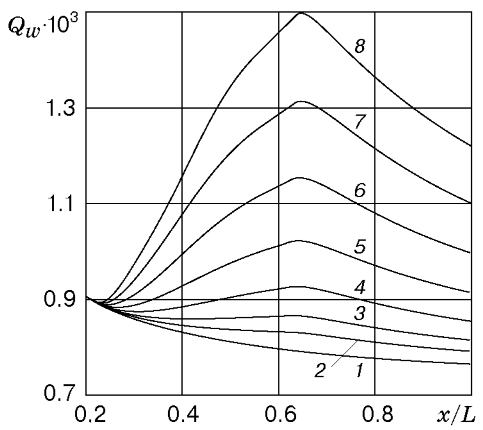

Fig. 3

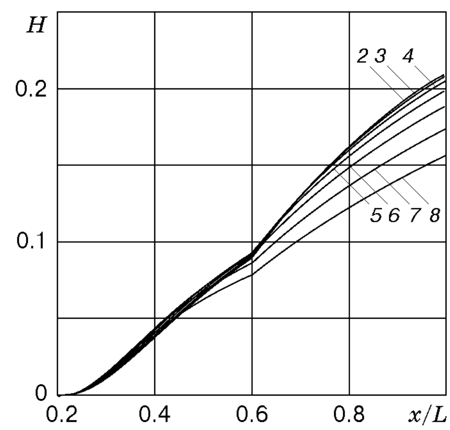

Fig. 4

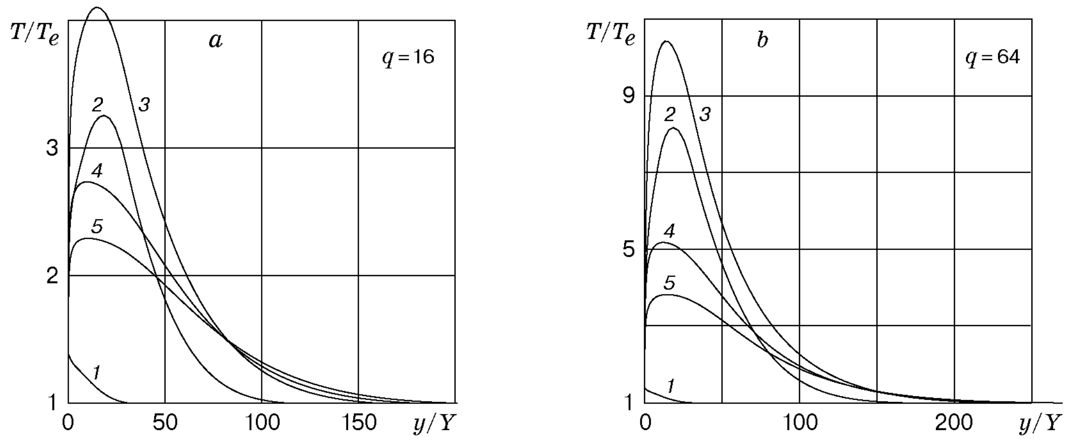

Fig. 5

values of the parameter c_f corresponding to the absence of heat addition are slowly recovered downstream from the thermal source.

The effect of heat-addition intensity on the coefficient of the total friction force $C_F = 2F_f\rho_e^{-1}u_e^{-2}(x - x_1)^{-1}$ is shown in Fig. 2 $\Big(F_f(x) = \displaystyle\int_{x_1}^{x} \tau_w\,dx,\ x_1 = 0.2L\Big)$. Thermal sources with the parameters $q = 16$ and 64 ensure a decrease in the friction force $F_f(0.6L)$ by 40 and 60%, respectively, which is much greater than the values calculated in [1] for a supersonic boundary layer on a thermally insulated surface.

Figure 3 shows the distribution of heat fluxes to the wall $Q_w = -2q_w\rho_e^{-1}u_e^{-3}$, where $q_w = -(\lambda_*\partial T/\partial y)_{y=0}$. Introducing into the flow a thermal source with a low value of specific heat addition ($q \leqslant 4$) does not lead to significant heating of the wall. With increasing specific heat addition, the heat flux from the gas to the wall significantly increases. Thus, for $q = 16$ and 64, the maximum values of heat fluxes are 1.5 and 1.9 times, respectively, greater than in the absence of the thermal action.

To evaluate the effectiveness of using the thermal source for decreasing the friction force, we use the parameter [4]

$$H = \frac{\Delta N}{Q_*}, \qquad \Delta N = \Delta F_f u_e, \qquad Q_* = \int_{x_1}^{x}\int_{0}^{y_e} \rho Q\,dy\,dx,$$

where $\Delta F_f = (F_f)_q - (F_f)_0$ and $Q_*(x)$ is the amount of external heat added to the gas upstream of the cross section x per unit time. If the friction force decreases by ΔF_f due to energy release, correspondingly, the power of the engine that ensures translational motion of the body with a velocity u_e may be reduced by ΔN. We note that the linear dependence of the total amount of heat $Q_*(0.6L)$ on the specific heat addition q, which is valid for low values of q, is violated with increasing this parameter. Thus, when q increases from

16 to 32, the total amount of heat increases only by a factor of 1.4. This is explained by the decrease in the mass flow in region (2.1).

The dependence $H(x/L)$ is plotted in Fig. 4. For moderate values of the parameter q, the efficiency of heat addition depends weakly on the total amount of heat added to the flow. With increasing parameter q, the efficiency of the thermal source noticeably decreases.

The temperature profiles in some cross sections of the boundary layer for $q = 16$ and $q = 64$ are plotted in Fig. 5. Curves 1–5 correspond to the cross sections $x/L = 0.2$, 0.4, 0.6, 0.8, and 1.0, respectively. The outer boundary of the heat-addition region for small $(x - x_1)/L$ is close to the boundary-layer edge. Further downstream, the temperature maximum moves away from the wall. When the temperature addition from outside is terminated, the gas in the boundary layer is gradually cooled. In the case $q = 16$, the temperature range is comparatively narrow: the maximum temperature is four times the gas temperature in the external flow (Fig. 5a). Vice versa, for $q = 64$ this ratio is more than ten (Fig. 5b). In this wide temperature range, the assumption of a constant heat capacity of the gas becomes invalid; the heat capacity is changed because of excitation of internal degrees of freedom of molecules and dissociation.

Thus, the results confirm the possibility of choosing the optimal amount of the added heat, which ensures a significant decrease in the friction force. With increasing heat addition above this optimal value, the heat-addition efficiency decreases, and the intensity of heat fluxes on the wall and the temperature in the boundary layer become too high.

REFERENCES

1. A. V. Kazakov, M. N. Kogan, and A. P. Kuryachii, "Effect of local heat addition to a turbulent boundary layer on friction" *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 48–56 (1997).
2. "Flow in a turbulent supersonic boundary layer with external heat addition," Report No. GR 4436, Inst. of Mech. at the Moscow State Univ. (1995).
3. T. Cebeci and A. M. O. Smith, *Analysis of Turbulent Boundary Layers*, Academic Press, New York (1974).
4. P. Yu. Georgievskii and V. A. Levin, "Changing of the flow regime around a sphere by a local energy action on the free stream," in: *Problems of Mechanics of Continuous Media* (collected scientific papers) [in Russian], Inst. of Automation and Control Processes, Far-East Div., Russ. Acad. of Sci., Vladivostok (1996), pp. 86–95.